Maxent Models and Discriminative Estimation

Generative vs. Discriminative models

borrowing from Christopher Manning

Introduction

- So far we’ve looked at “generative models”
  - Language models, Naive Bayes
- But there is now much use of conditional or discriminative probabilistic models in NLP, Speech, IR (and ML generally)
- Because:
  - They give high accuracy performance
  - They make it easy to incorporate lots of linguistically important features

Joint vs. Conditional Models

- We have some data {(*d*, *c*)} of paired observations *d* and hidden classes *c*.
- Joint (generative) models place probabilities over both observed data and the hidden stuff (gene-rate the observed data from hidden stuff):
  - All the classic StatNLP models:
    - *n*-gram models, Naive Bayes classifiers, hidden Markov models, probabilistic context-free grammars, IBM machine translation alignment models

$P(c,d)$

Joint vs. Conditional Models

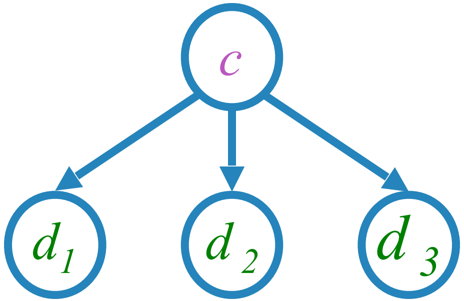
- Discriminative (conditional) models take the data as given, and put a probability over hidden structure given the data:

  $P(c|d)$

  - Logistic regression, conditional loglinear or maximum entropy models, conditional random fields
  - Also, SVMs, (averaged) perceptron, etc. are discriminative classifiers (but not directly probabilistic)

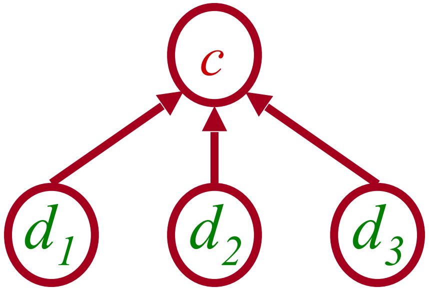
Bayes Net/Graphical Models

- Bayes net diagrams draw circles for random variables, and lines for direct dependencies
- Some variables are observed; some are hidden
- Each node is a little classifier (conditional probability table) based on incoming arcs



Naive Bayes

Generative



Logistic Regression

Discriminative

Conditional vs. Joint Likelihood

- A *joint* model gives probabilities $P(d,c)$ and tries to maximize this joint likelihood.
  - usually, it's trivial to choose weights: just relative frequencies.
- A *conditional* model gives probabilities $P(c|d)$. It takes the data as given and models only the conditional probability of the class.
  - We seek to maximize conditional likelihood.
  - Harder to do (as we'll see…)
  - More closely related to classification error.

Conditional models work well: Word Sense Disambiguation

Training Set	
Objective	Accuracy
Joint Like.	86.8
Cond. Like.	98.5

Test Set	
Objective	Accuracy
Joint Like.	73.6
Cond. Like.	76.1

(Klein and Manning 2002, using Senseval-1 Data)

- Even with exactly the same features, changing from joint to conditional estimation increases performance
- That is, we use the same smoothing, and the same word-class features, we just change the numbers (parameters)

Maxent Models and Discriminative Estimation

Generative vs. Discriminative models

borrowing from Christopher Manning

Discriminative Model Features

Making features from text for discriminative NLP models

borrowing from Christopher Manning

Features

- In these slides and most maxent work: *features* f are elementary pieces of evidence that link aspects of what we observe *d* with a category *c* that we want to predict
- A feature is a function with a bounded real value

Example features

- $f_1(c, d) \equiv [c = \text{LOCATION} \wedge w_{-1} = \text{“in”} \wedge \text{isCapitalized}(w)]$
- $f_2(c, d) \equiv [c = \text{LOCATION} \wedge \text{hasAccentedLatinChar}(w)]$
- $f_3(c, d) \equiv [c = \text{DRUG} \wedge \text{ends}(w, \text{“c”})]$

- Models will assign to each feature a *weight:*
  - A positive weight votes that this configuration is likely correct
  - A negative weight votes that this configuration is likely incorrect

Feature Expectations

- We will crucially make use of two *expectations*
  - actual or predicted counts of a feature firing:

- Empirical count (expectation) of a feature:

$$\text{empirical } E(f_i) = \sum_{(c,d)\in \text{observed}(C,D)} f_i(c,d)$$

- Model expectation of a feature:

$$E(f_i) = \sum_{(c,d)\in (C,D)} P(c,d)\, f_i(c,d)$$

Features

- In NLP uses, usually a feature specifies
  1. an indicator function – a yes/no boolean matching function – of properties of the input and
  2. a particular class

$$f_i(c, d) \equiv [\Phi(d) \wedge c = c_j]$$

[Value is 0 or 1]

- Each feature picks out a data subset and suggests a label for it

Feature-Based Models

- The decision about a data point is based only on the features active at that point.

Data
BUSINESS: Stocks hit a yearly low …

Label: BUSINESS
Features
{…, stocks, hit, a, yearly, low, …}

Text Categorization

Data
… to restructure bank:MONEY debt.

Label: MONEY
Features
{…, w_{-1}=restructure, w_{+1}=debt, …}

Word-Sense Disambiguation

Data
DT JJ NN …
The previous fall …

Label: NN
Features
{w=fall, t_{-1}=JJ w_{-1}=previous}

POS Tagging

Example: Text Categorization

(Zhang and Oles 2001)

- Features are presence of each word in a document and the document class (they do feature selection to use reliable indicator words)
- Tests on classic Reuters data set (and others)
  - Naïve Bayes: 77.0% F_1
  - Linear regression: 86.0%
  - Logistic regression: 86.4%
  - Support vector machine: 86.5%
- Paper emphasizes the importance of *regularization* (smoothing) for successful use of discriminative methods (not used in much early NLP/IR work)

Other Maxent Classifier Examples

- You can use a maxent classifier whenever you want to assign data points to one of a number of classes:
  - Sentence boundary detection (Mikheev 2000)
    - Is a period end of sentence or abbreviation?
  - Sentiment analysis (Pang and Lee 2002)
    - Word unigrams, bigrams, POS counts, …
  - PP attachment (Ratnaparkhi 1998)
    - Attach to verb or noun? Features of head noun, preposition, etc.
  - Parsing decisions in general (Ratnaparkhi 1997; Johnson et al. 1999, etc.)

Discriminative Model Features

Making features from text for discriminative NLP models

borrowing from Christopher Manning

Feature-based Linear Classifiers

How to put features into a classifier

borrowing from Christopher Manning

Feature-Based Linear Classifiers

- Linear classifiers at classification time:
  - Linear function from feature sets $\{f_i\}$ to classes $\{c\}$.
  - Assign a weight λ_i to each feature f_i.
  - We consider each class for an observed datum d
  - For a pair (c,d), features vote with their weights:
    - vote(c) = $\Sigma\lambda_i f_i(c,d)$

PERSON *in Québec*

LOCATION *in Québec*

DRUG *in Québec*

  - Choose the class c which maximizes $\Sigma\lambda_i f_i(c,d)$

Feature-Based Linear Classifiers

- Linear classifiers at classification time:
  - Linear function from feature sets $\{f_i\}$ to classes $\{c\}$.
  - Assign a weight λ_i to each feature f_i.
  - We consider each class for an observed datum d
  - For a pair (c,d), features vote with their weights:
    - $\text{vote}(c) = \Sigma\lambda_i f_i(c,d)$

  - Choose the class c which maximizes $\Sigma\lambda_i f_i(c,d)$ = LOCATION

Feature-Based Linear Classifiers

There are many ways to chose weights for features

- Perceptron: find a currently misclassified example, and nudge weights in the direction of its correct classification

- Margin-based methods (Support Vector Machines)

Feature-Based Linear Classifiers

- Exponential (log-linear, maxent, logistic, Gibbs) models:
  - Make a probabilistic model from the linear combination $\Sigma\lambda_i f_i(c,d)$

$$P(c|d,\lambda) = \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

Makes votes positive (numerator)

Normalizes votes (denominator)

- P(LOCATION|*in Québec*) = $e^{1.8}e^{-0.6}/(e^{1.8}e^{-0.6} + e^{0.3} + e^{0}) = 0.586$
- P(DRUG|*in Québec*) = $e^{0.3} /(e^{1.8}e^{-0.6} + e^{0.3} + e^{0}) = 0.238$
- P(PERSON|*in Québec*) = $e^{0} /(e^{1.8}e^{-0.6} + e^{0.3} + e^{0}) = 0.176$

- The weights are the parameters of the probability model, combined via a “soft max” function

Feature-Based Linear Classifiers

- Exponential (log-linear, maxent, logistic, Gibbs) models:
  - Given this model form, we will choose parameters $\{\lambda_i\}$ that *maximize the conditional likelihood* of the data according to this model.
  - We construct not only classifications, but probability distributions over classifications.
    - There are other (good!) ways of discriminating classes – SVMs, boosting, even perceptrons – but these methods are not as trivial to interpret as distributions over classes.

Aside: logistic regression

- Maxent models in NLP are essentially the same as multiclass logistic regression models in statistics (or machine learning)
  - If you haven't seen these before, don't worry, this presentation is self-contained!
  - If you have seen these before you might think about:
    - The parameterization is slightly different in a way that is advantageous for NLP-style models with tons of sparse features (but statistically inelegant)
    - The key role of feature functions in NLP and in this presentation
      - The features are more general, with f also being a function of the class

Quiz Question

- Assuming exactly the same set up (3 class decision: LOCATION, PERSON, or DRUG; 3 features as before, maxent), what are:
  - P(PERSON | *by Goéric*) =
  - P(LOCATION | *by Goéric*) =
  - P(DRUG | *by Goéric*) =

- 1.8 $f_1(c, d) \equiv [c = \text{LOCATION} \wedge w_{-1} = \text{“in”} \wedge \text{isCapitalized}(w)]$
- -0.6 $f_2(c, d) \equiv [c = \text{LOCATION} \wedge \text{hasAccentedLatinChar}(w)]$
- 0.3 $f_3(c, d) \equiv [c = \text{DRUG} \wedge \text{ends}(w, \text{“c”})]$

PERSON
by Goéric

LOCATION
by Goéric

DRUG
by Goéric

$$P(c \mid d, \lambda) = \frac{\exp \sum_i \lambda_i f_i(c, d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c', d)}$$

Feature-based Linear Classifiers

How to put features into a classifier

borrowing from Christopher Manning

Building a Maxent Model

The nuts and bolts

borrowing from Christopher Manning

Building a Maxent Model

- We define features (indicator functions) over data points
  - Features represent sets of data points which are distinctive enough to deserve model parameters.
    - Words, but also “word contains number”, “word ends with *ing*”, etc.
- We will simply encode each Φ feature as a unique String
  - A datum will give rise to a set of Strings: the active Φ features
  - Each feature $f_i(c, d) \equiv [\Phi(d) \wedge c = c_j]$ gets a real number weight
- We concentrate on Φ features but the math uses i indices of f_i

Building a Maxent Model

- Features are often added during model development to target errors
  - Often, the easiest thing to think of are features that mark bad combinations
- Then, for any given feature weights, we want to be able to calculate:
  - Data conditional likelihood
  - Derivative of the likelihood wrt each feature weight
    - Uses expectations of each feature according to the model
- We can then find the optimum feature weights (discussed later).

Building a Maxent Model

The nuts and bolts

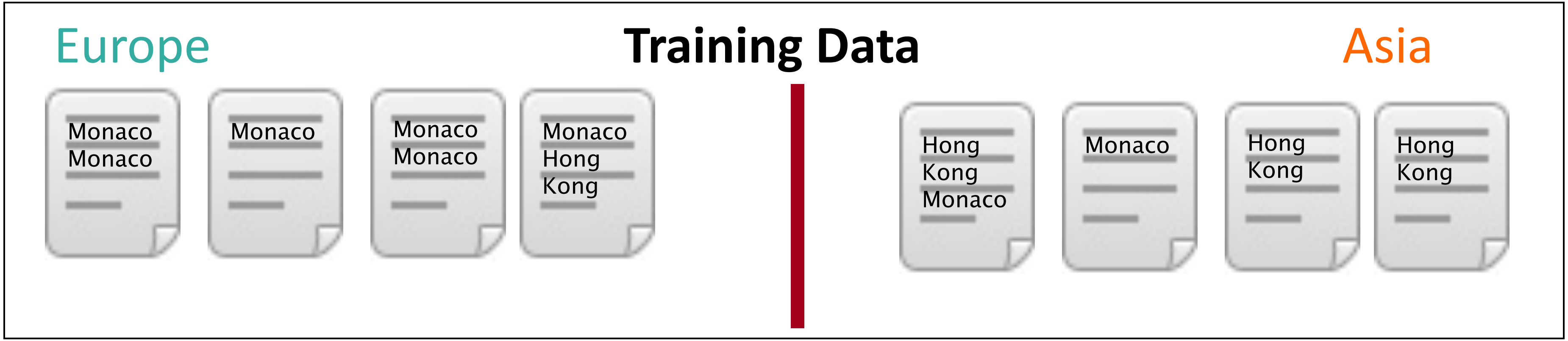
borrowing from Christopher Manning

Naive Bayes vs. Maxent models

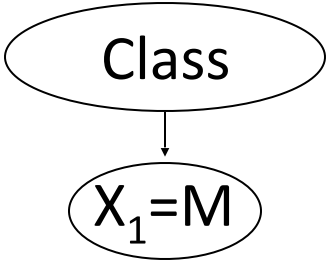
Generative vs. Discriminative models: The problem of overcounting evidence

borrowing from Christopher Manning

Text classification: Asia or Europe



NB Model



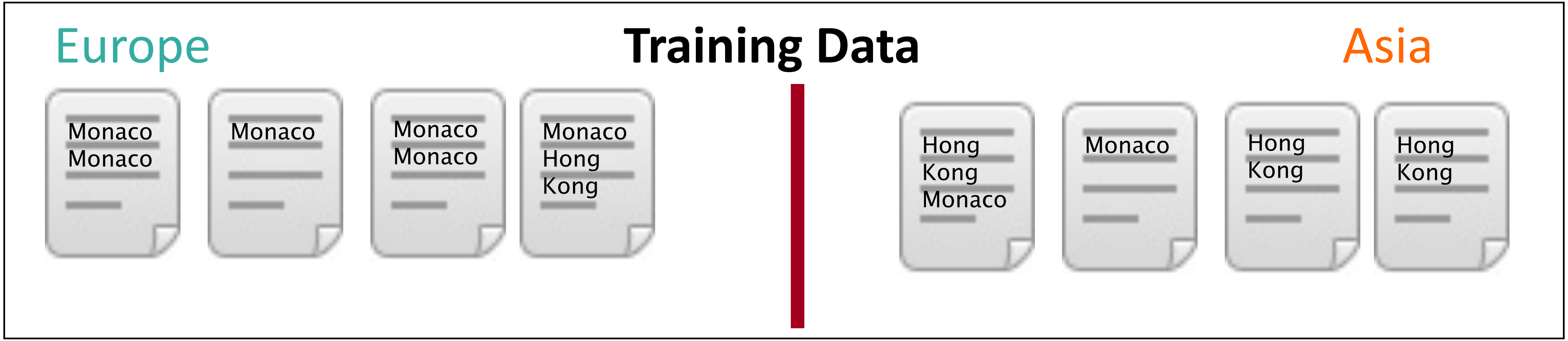
NB FACTORS:

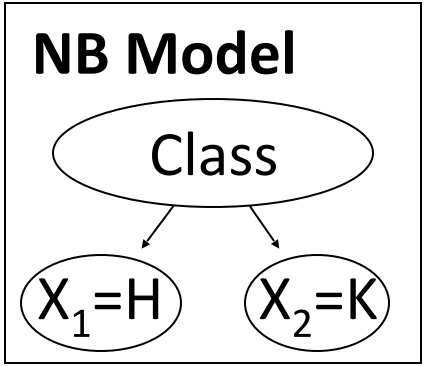
- P(A) = P(E) =
- P(M|A) =
- P(M|E) =

PREDICTIONS:

- P(A,M) =
- P(E,M) =
- P(A|M) =
- P(E|M) =

Text classification: Asia or Europe





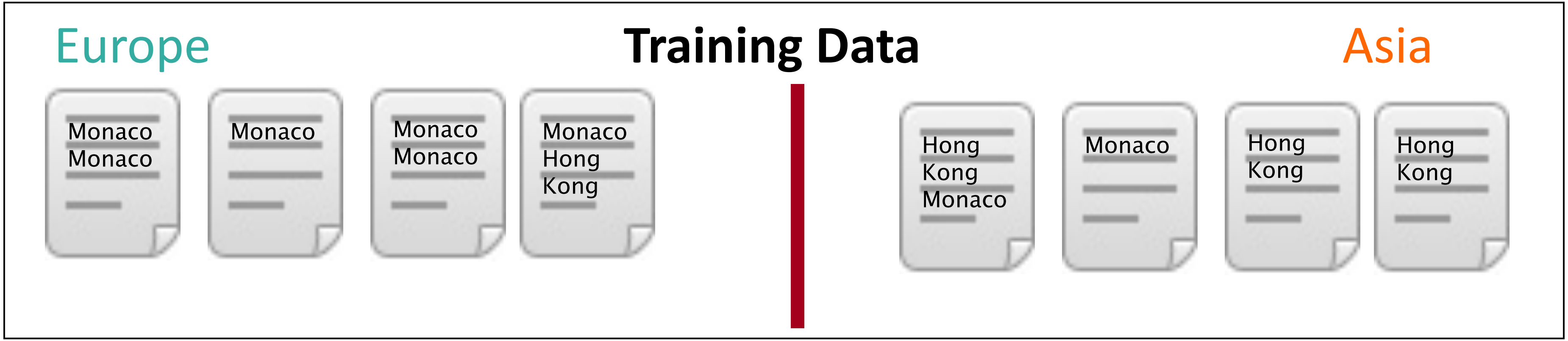
NB FACTORS:

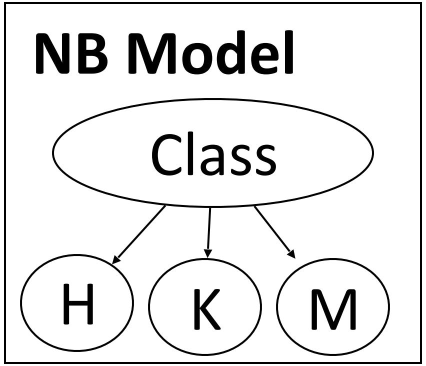
- P(A) = P(E) =
- P(H|A) = P(K|A) =
- P(H|E) = PK|E) =

PREDICTIONS:

- P(A,H,K) =
- P(E,H,K) =
- P(A|H,K) =
- P(E|H,K) =

Text classification: Asia or Europe





NB FACTORS:

- P(A) = P(E) =
- P(M|A) =
- P(M|E) =
- P(H|A) = P(K|A) =
- P(H|E) = PK|E) =

PREDICTIONS:

- P(A,H,K,M) =
- P(E,H,K,M) =
- P(A|H,K,M) =
- P(E|H,K,M) =

Naive Bayes vs. Maxent Models

- Naive Bayes models multi-count correlated evidence
  - Each feature is multiplied in, even when you have multiple features telling you the same thing
- Maximum Entropy models (pretty much) solve this problem
  - As we will see, this is done by weighting features so that model expectations match the observed (empirical) expectations

Naive Bayes vs. Maxent models

Generative vs. Discriminative models: The problem of overcounting evidence

borrowing from Christopher Manning

Maxent Models and Discriminative Estimation

Maximizing the likelihood

borrowing from Christopher Manning

Exponential Model Likelihood

- Maximum (Conditional) Likelihood Models :
  - Given a model form, choose values of parameters to maximize the (conditional) likelihood of the data.

$$\log P(C \mid D, \lambda) = \sum_{(c,d)\in(C,D)} \log P(c \mid d, \lambda) = \sum_{(c,d)\in(C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

The Likelihood Value

- The (log) conditional likelihood of iid data (*C*,*D*) according to maxent model is a function of the data and the parameters λ:

$$\log P(C \mid D,\lambda) = \log \prod_{(c,d)\in(C,D)} P(c \mid d,\lambda) = \sum_{(c,d)\in(C,D)} \log P(c \mid d,\lambda)$$

- If there aren't many values of *c*, it's easy to calculate:

$$\log P(C \mid D,\lambda) = \sum_{(c,d)\in(C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

The Likelihood Value

- We can separate this into two components:

$$\log P(C|D,\lambda) = \sum_{(c,d)\in(C,D)} \log \exp \sum_i \lambda_i f_i(c,d) - \sum_{(c,d)\in(C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)$$

$$\log P(C|D,\lambda) = N(\lambda) - M(\lambda)$$

- The derivative is the difference between the derivatives of each component

The Derivative I: Numerator

$$\frac{\partial N(\lambda)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \log \exp \sum_i \lambda_{ci} f_i(c,d)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{\partial \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} f_i(c,d)$$

Derivative of the numerator is: the empirical count(f_i, c)

The Derivative II: Denominator

$$\frac{\partial M(\lambda)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \frac{\partial \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c',d)}{1} \frac{\partial \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c',d)}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \frac{\partial \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \sum_{c'} P(c' \mid d, \lambda)\ f_i(c',d)$$

= predicted count(f_i, λ)

The Derivative III

$$\frac{\partial \log P(C \mid D, \lambda)}{\partial \lambda_i} = \text{actual count}(f_i, C) - \text{predicted count}(f_i, \lambda)$$

- The optimum parameters are the ones for which each feature's predicted expectation equals its empirical expectation. The optimum distribution is:
  - Always unique (but parameters may not be unique)
  - Always exists (if feature counts are from actual data).

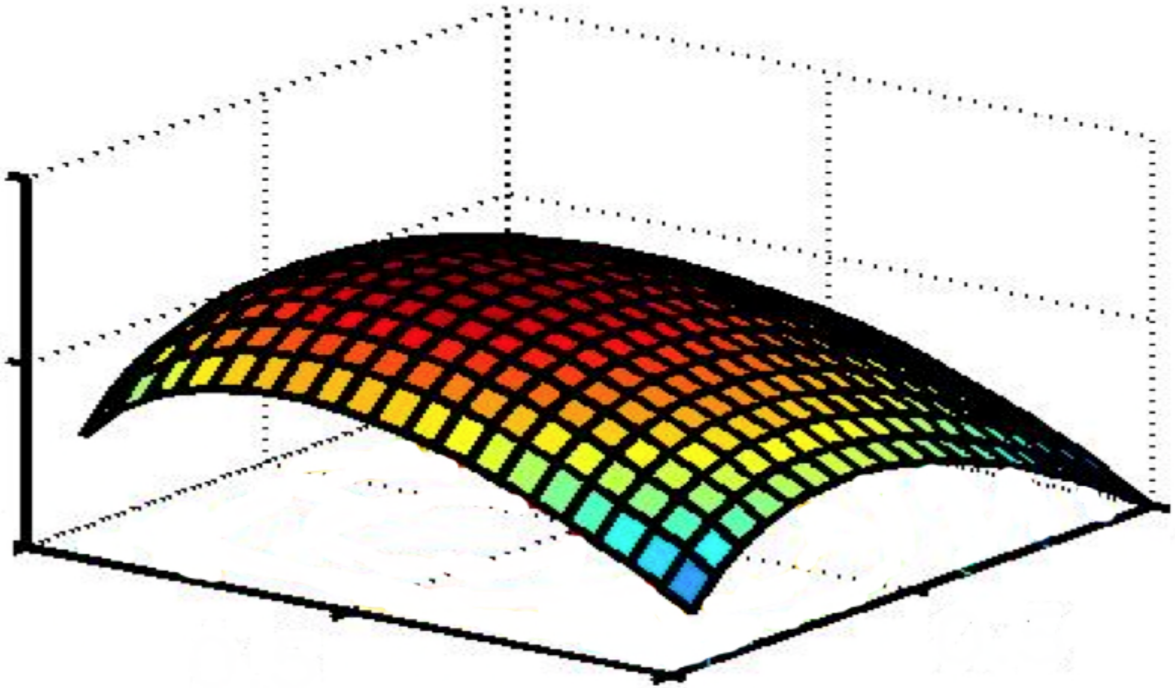
Finding the optimal parameters

- We want to choose parameters $\lambda_1, \lambda_2, \lambda_3, \ldots$ that maximize the conditional log-likelihood of the training data

$$CLogLik(D) = \sum_{i=1}^{n} \log P(c_i \mid d_i)$$

- To be able to do that, we've worked out how to calculate the function value and its partial derivatives (its gradient)

A likelihood surface



Finding the optimal parameters

- Use your favorite numerical optimization package….
  - Commonly, you **minimize** the negative of *CLogLik*

1. Gradient descent (GD); Stochastic gradient descent (SGD)
2. Iterative proportional fitting methods: Generalized Iterative Scaling (GIS) and Improved Iterative Scaling (IIS)
3. Conjugate gradient (CG), perhaps with preconditioning
4. Quasi-Newton methods – limited memory variable metric (LMVM) methods, in particular, L-BFGS

Maxent Models and Discriminative Estimation

Maximizing the likelihood

borrowing from Christopher Manning