## Topic models (Latent Dirichlet Allocation)

Klinton Bicknell

borrowing from Roger Levy, Tom Griffiths, David Blei

### **Topic modeling - Motivation**

Topic modeling provides methods for automatically organizing, understanding, searching, and summarizing large electronic archives.

- 1 Uncover the hidden topical patterns that pervade the collection.
- 2 Annotate the documents according to those topics.
- **3** Use the annotations to organize, summarize, and search the texts.

### Discover topics from a corpus

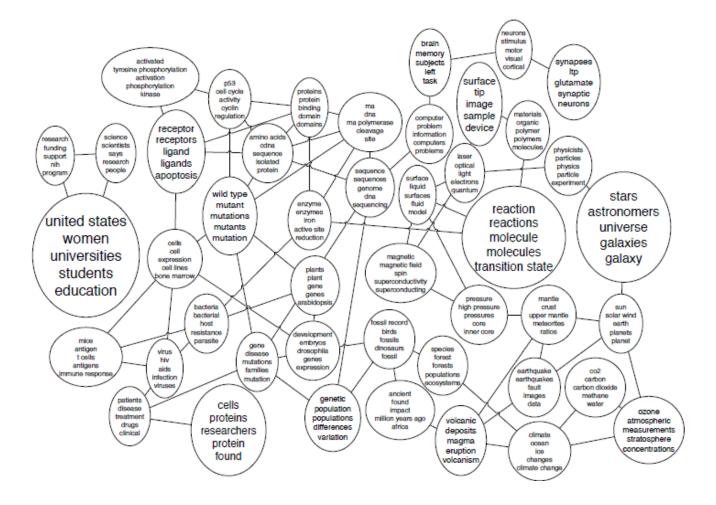
human genome dna genetic genes sequence gene molecular sequencing map information genetics mapping project sequences

evolution evolutionary species organisms life origin biology groups phylogenetic living diversity group new two common

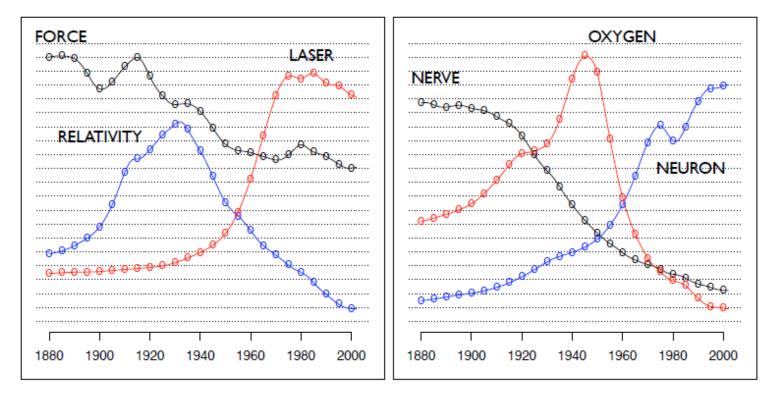
disease host bacteria diseases resistance bacterial new strains control infectious malaria parasite parasites united tuberculosis

computer models information data computers system network systems model parallel methods networks software new simulations

### Model connections between topics



### Model the evolution of topics over time (or other relevant variable)



"Theoretical Physics"

"Neuroscience"

# Connection to ML research

From a machine learning perspective, topic modeling is a case study in applying hierarchical Bayesian models to grouped data, like documents or images. Topic modeling research touches on

- Directed graphical models
- Conjugate priors and nonconjugate priors
- Time series modeling
- Modeling with graphs
- Hierarchical Bayesian methods
- Fast approximate posterior inference (MCMC, variational methods)
- Exploratory data analysis
- Model selection and nonparametric Bayesian methods
- Mixed membership models

# Intuition behind LDA

### Seeking Life's Bare (Genetic) Necessities

Heemophilus

**behome** 

COLD SPRING HARBOR, NEW YORK— How many genes does an organism need to survive? Last week at the genome meeting here,<sup>®</sup> two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The

other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

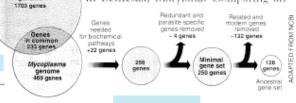
Although the numbers don't match precisely, those predictions

\* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.

SCIENCE • VOL. 272 • 24 MAY 1996

"are not all that far apart," especially in comparison to the 75,000 genes in the human genome, notes Siv Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic numbers game, particularly as more and more genomes are completely mapped and sequenced. "It may be a way of organizing any newly sequenced genome," explains

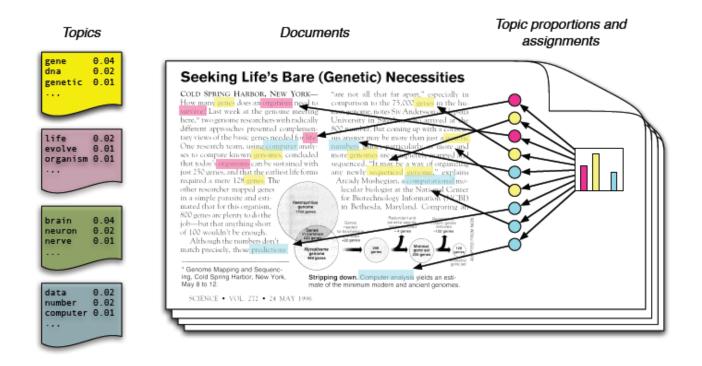
Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an



Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.

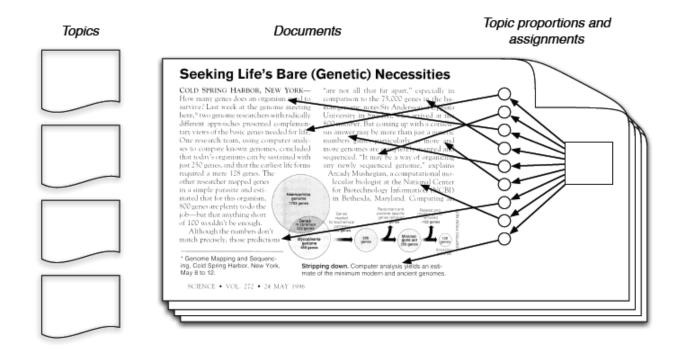
### Simple intuition: Documents exhibit multiple topics.

# Generative model



- Each document is a random mixture of corpus-wide topics
- Each word is drawn from one of those topics

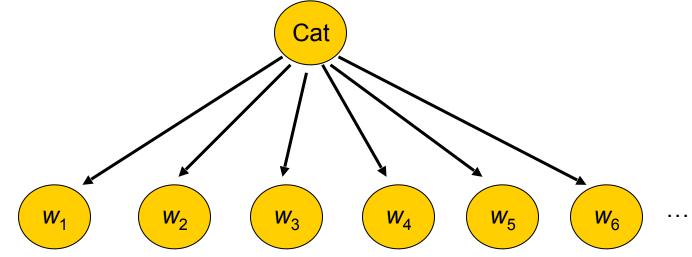
# The posterior distribution



- In reality, we only observe the documents
- Our goal is to infer the underlying topic structure

# Previously

- Supervised text categorization through Naïve Bayes
- Generative model: first generate a document category, then words in the document (unigram model)



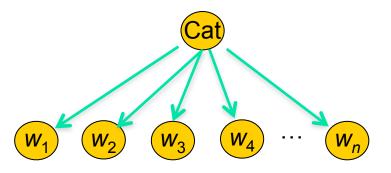
• Inference: obtain posterior over document categories using Bayes rule (argmax to choose the category)  $P(Cat | w_{1...n}) = \frac{P(w_{1...n} | Cat)P(Cat)}{P(w_{1...n})}$ 

# What we're doing here

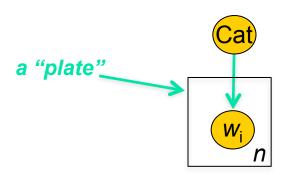
- Supervised categorization requires hand-labeling documents
- This can be extremely time-consuming
- Unlabeled documents are cheap
- So we'd really like to do *unsupervised* text categorization
- Now we'll look at unsupervised learning within the Naïve Bayes model

### Compact graphical model representations

We're going to lean heavily on graphical model representations here.

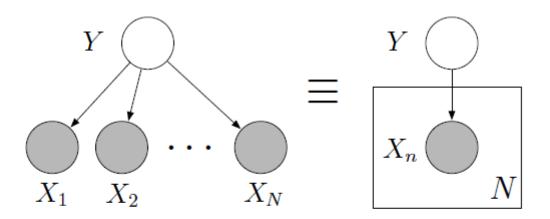


• We'll use a more compact notation:



"generate a word from Cat n times"

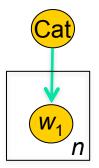
# Graphical models (Aside)



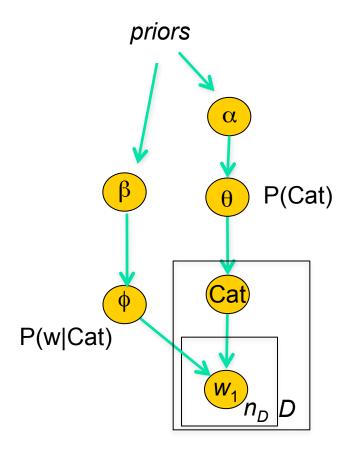
- Nodes are random variables
- Edges denote possible dependence
- Observed variables are shaded
- Plates denote replicated structure
- Structure of the graph defines the pattern of conditional dependence between the ensemble of random variables
- E.g., this graph corresponds to

$$p(y, x_1, ..., x_N) = p(y) \prod_{n=1}^N p(x_n | y)$$

- Now suppose that Cat isn't observed
- We need to learn two distributions:
  - P(Cat)
  - P(wlCat)
- How do we do this?
  - We might use the method of maximum likelihood (MLE)
  - But it turns out that the likelihood surface is highly nonconvex and lots of information isn't contained in a point estimate
  - Alternative: Bayesian methods



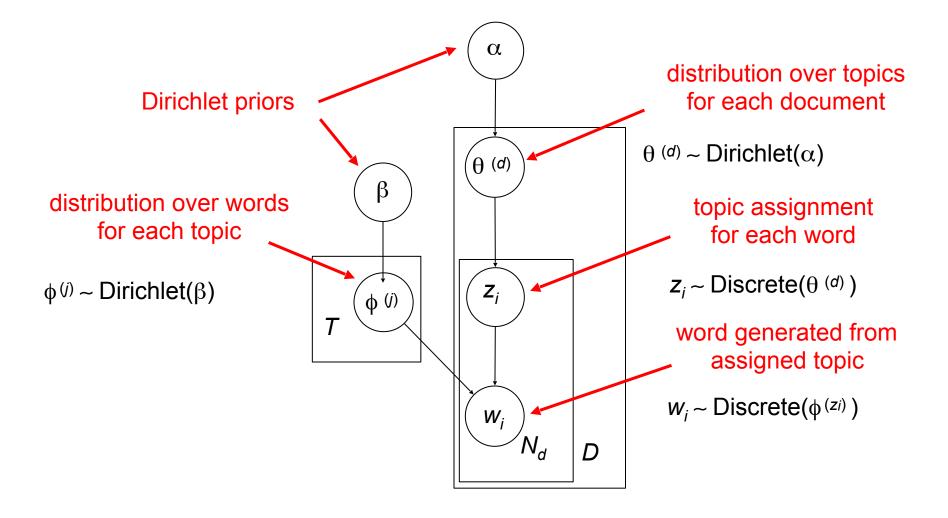
### **Bayesian document categorization**



### Latent Dirichlet allocation

(Blei, Ng, & Jordan, 2001; 2003)

Main difference: one topic per word



### A generative model for documents

W	<i>P</i> ( <i>w</i>  Cat = 1)	W	<i>P</i> ( <i>w</i>  Cat = 2)
HEART LOVE SOUL TEARS	0.2 0.2 0.2 0.2	HEART LOVE SOUL TEARS	0.0 0.0 0.0 0.0
JOY SCIENTIFIC KNOWLEDG WORK RESEARCH MATHEMATI	0.0 0.0 0.0 0.0 0.0 0.0	JOY SCIENTIFIC KNOWLED WORK RESEARCI MATHEMA	0.0 C 0.2 GE 0.2 0.2 H 0.2

Choose mixture weights for each document, generate "bag of words"

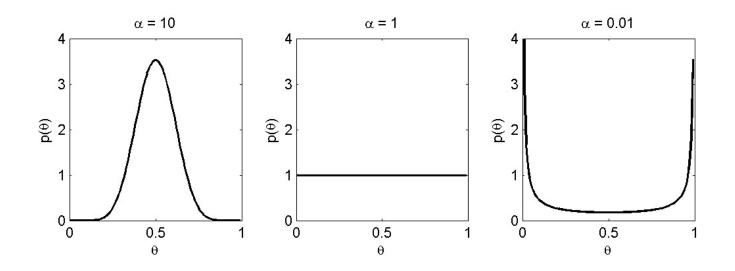
 $\{P(z = 1), P(z = 2)\}$ 

{0, 1}	MATHEMATICS KNOWLEDGE RESEARCH WORK MATHEMATICS RESEARCH WORK SCIENTIFIC MATHEMATICS WORK
{0.25, 0.75}	SCIENTIFIC KNOWLEDGE MATHEMATICS SCIENTIFIC HEART LOVE TEARS KNOWLEDGE HEART
{0.5, 0.5}	MATHEMATICS HEART RESEARCH LOVE MATHEMATICS WORK TEARS SOUL KNOWLEDGE HEART
{0.75, 0.25}	WORK JOY SOUL TEARS MATHEMATICS TEARS LOVE LOVE SOUL
{1, 0}	TEARS LOVE JOY SOUL LOVE TEARS SOUL SOUL TEARS JOY

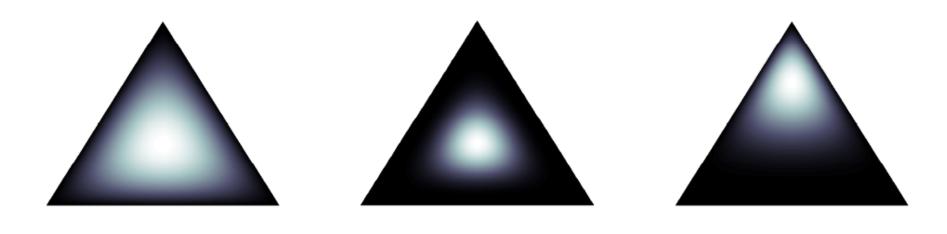
Multivariate equivalent of Beta distribution

$$p(\theta \mid Cat) = \frac{\Gamma(T\alpha)}{\Gamma(\alpha)^T} \prod_{j=1}^T \theta_j^{\alpha-1}$$

- Hyperparameters  $\boldsymbol{\alpha}$  determine form of the prior



# **Dirichlet Examples**

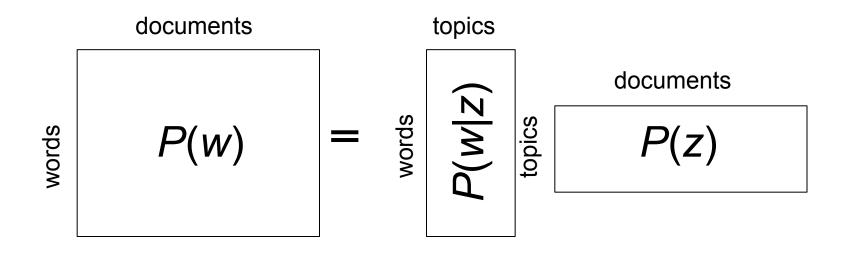


$\alpha = (2, 2, 2)$	$\alpha = (5, 5, 5)$	$\alpha = (2, 2, 25)$
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### Darker implies lower magnitude

\alpha < 1 leads to sparser topics

## Matrix factorization interpretation



Maximum-likelihood estimation is finding the factorization that minimizes KL divergence

(Hofmann, 1999)

# Interpretable topics

DISEASE	WATER	MIND	STORY	FIELD	SCIENCE	BALL	JOB
BACTERIA	FISH	WORLD	STORIES	MAGNETIC	STUDY	GAME	WORK
DISEASES	SEA	DREAM	TELL	MAGNET	SCIENTISTS	TEAM	JOBS
GERMS	SWIM	DREAMS	CHARACTER	WIRE	SCIENTIFIC	FOOTBALL	CAREER
FEVER	SWIMMING	THOUGHT	CHARACTERS	NEEDLE	KNOWLEDGE	BASEBALL	EXPERIENCE
CAUSE	POOL	IMAGINATION	AUTHOR	CURRENT	WORK	PLAYERS	EMPLOYMENT
CAUSED	LIKE	MOMENT	READ	COIL	RESEARCH	PLAY	OPPORTUNITIES
SPREAD	SHELL	THOUGHTS	TOLD	POLES	CHEMISTRY	FIELD	WORKING
VIRUSES	SHARK	OWN	SETTING	IRON	TECHNOLOGY	PLAYER	TRAINING
INFECTION	TANK	REAL	TALES	COMPASS	MANY I	BASKETBAL	
VIRUS	SHELLS	LIFE	PLOT	LINES	MATHEMATICS		CAREERS
MICROORGANISM		IMAGINE	TELLING	CORE	BIOLOGY	PLAYED	POSITIONS
PERSON	DIVING	SENSE	SHORT	ELECTRIC	FIELD	PLAYING	FIND
INFECTIOUS	DOLPHINS	CONSCIOUSNES	S FICTION	DIRECTION	PHYSICS	HIT	POSITION
COMMON	SWAM	STRANGE	ACTION	FORCE	LABORATORY	TENNIS	FIELD
CAUSING	LONG	FEELING	TRUE	MAGNETS	STUDIES	TEAMS	OCCUPATIONS
SMALLPOX	SEAL	WHOLE	EVENTS	BE	WORLD	GAMES	REQUIRE
BODY	DIVE	BEING	TELLS	MAGNETISM		SPORTS	OPPORTUNITY
INFECTIONS	DOLPHIN	MIGHT	TALE	POLE	STUDYING	BAT	EARN
CERTAIN	UNDERWATER	HOPE	NOVEL	INDUCED	SCIENCES	TERRY	ABLE

each column shows words from a single topic, ordered by P(w|z)

# Handling multiple senses

DISEASE	WATER	MIND	STORY	FIELD	SCIENCE	BALL	JOB
BACTERIA	FISH	WORLD	STORIES	MAGNETIC	STUDY	GAME	WORK
DISEASES	SEA	DREAM	TELL	MAGNET	SCIENTISTS	TEAM	JOBS
GERMS	SWIM	DREAMS	CHARACTER	WIRE	SCIENTIFIC	FOOTBALL	CAREER
FEVER	SWIMMING	THOUGHT	CHARACTERS	NEEDLE	KNOWLEDGE	BASEBALL	EXPERIENCE
CAUSE	POOL	IMAGINATION	AUTHOR	CURRENT	WORK	PLAYERS	EMPLOYMENT
CAUSED	LIKE	MOMENT	READ	COIL	RESEARCH	PLAY	OPPORTUNITIES
SPREAD	SHELL	THOUGHTS	TOLD	POLES	CHEMISTRY	FIELD	WORKING
VIRUSES	SHARK	OWN	SETTING	IRON	TECHNOLOGY	PLAYER	TRAINING
INFECTION	TANK	REAL	TALES	COMPASS	MANY	BASKETBAL	
VIRUS	SHELLS	LIFE	PLOT	LINES	MATHEMATICS		CAREERS
MICROORGANISM	IS SHARKS	IMAGINE	TELLING	CORE	BIOLOGY	PLAYED	POSITIONS
PERSON	DIVING	SENSE	SHORT	ELECTRIC	FIELD	PLAYING	FIND
INFECTIOUS	DOLPHINS	CONSCIOUSNES	S FICTION	DIRECTION	PHYSICS	HIT	POSITION
COMMON	SWAM	STRANGE	ACTION	FORCE	LABORATORY	TENNIS	FIELD
CAUSING	LONG	FEELING	TRUE	MAGNETS	STUDIES	TEAMS	OCCUPATIONS
SMALLPOX	SEAL	WHOLE	EVENTS	BE	WORLD	GAMES	REQUIRE
BODY	DIVE	BEING	TELLS	MAGNETISM		SPORTS	OPPORTUNITY
INFECTIONS	DOLPHIN	MIGHT	TALE	POLE	STUDYING	BAT	EARN
CERTAIN	UNDERWATER	HOPE	NOVEL	INDUCED	SCIENCES	TERRY	ABLE

each column shows words from a single topic, ordered by P(w|z)

### Explore and browse document collections

### Chance and Statistical Significance in Protein and DNA Sequence Analysis

### Samuel Karlin and Volker Brendel

Тор wo	ords from the t	Expected topic proportions		
sequence	measured	residues	computer	
region	average	binding	methods	
pcr	range	domains	number	
identified	values	helix	two	
fragments	different	cys	principle	
two	size	regions	design	
genes	three	structure	access	
three	calculated	terminus	processing	
cdna	two	terminus	advantage	
analysis	low	site	important	

### Abstract with the most likely topic assignments

Statistical approaches help in the determination of significant configurations in protein and nucleic acid sequence data. Three recent statistical methods are discussed: (i) scorebased sequence analysis that provides a means for characterizing anomalies in local sequence text and for evaluating sequence comparisons; (ii) quantile distributions of amino acid usage that reveal general compositional biases in proteins and evolutionary relations; and (ii) r-scan statistics that can be applied to the analysis of spacings of sequence markers.

### Top Ten Similar Documents

Exhaustive Matching of the Entire Protein Sequence Database How Big Is the Universe of Exons? Counting and Discounting the Universe of Exons Detecting Subtle Sequence Signals: A Gibbs Sampling Strategy for Multiple Alignment Ancient Conserved Regions in New Gene Sequences and the Protein Databases A Method to Identify Protein Sequences that Fold into a Known Three- Dimensional Structure Testing the Exon Theory of Genes: The Evidence from Protein Structure Predicting Coiled Coils from Protein Sequences Genome Sequence of the Nematode C. elegans: A Platform for Investigating Biology

# Why does LDA "work" ?

Why does the LDA posterior put "topical" words together?

- Word probabilities are maximized by dividing the words among the topics. (More terms means more mass to be spread around.)
- In a mixture, this is enough to find clusters of co-occurring words.
- In LDA, the Dirichlet on the topic proportions can encourage sparsity, i.e., a document is penalized for using many topics.
- Loosely, this can be thought of as softening the strict definition of "co-occurrence" in a mixture model.
- This flexibility leads to sets of terms that more tightly co-occur.

## Inverting the generative model

- Maximum a posteriori estimation (EM)
  - e.g., Hofmann (1999)
- Deterministic approximate algorithms
  - variational EM; Blei, Ng & Jordan (2001; 2003)
  - expectation propagation; Minka & Lafferty (2002)
- Markov chain Monte Carlo
  - full Gibbs sampler; Pritchard et al. (2000)
  - collapsed Gibbs sampler; Griffiths & Steyvers (2004)

## The collapsed Gibbs sampler

 Using conjugacy of Dirichlet and multinomial distributions, integrate out continuous parameters

$$P(\mathbf{z}) = \int_{\Delta_T^D} P(\mathbf{z} \mid \Theta) p(\Theta) d\Theta = \prod_{d=1}^D \frac{\prod_j \Gamma(n_j^{(d)} + \alpha)}{\Gamma(\alpha)^T} \frac{\Gamma(T\alpha)}{\Gamma(\sum_j n_j^{(d)} + \alpha)}$$
$$P(\mathbf{w} \mid \mathbf{z}) = \int_{\Delta_W^T} P(\mathbf{w} \mid \mathbf{z}, \Phi) p(\Phi) d\Phi = \prod_{j=1}^T \frac{\prod_w \Gamma(n_w^{(j)} + \beta)}{\Gamma(\beta)^W} \frac{\Gamma(W\beta)}{\Gamma(\sum_w n_w^{(j)} + \beta)}$$

Defines a distribution on discrete ensembles z

$$P(\mathbf{z} \mid \mathbf{w}) = \frac{P(\mathbf{w} \mid \mathbf{z})P(\mathbf{z})}{\sum_{\mathbf{z}} P(\mathbf{w} \mid \mathbf{z})P(\mathbf{z})}$$

### The collapsed Gibbs sampler

Sample each z<sub>i</sub> conditioned on z<sub>i</sub>

$$P(z_i \mid \mathbf{w}, \mathbf{z}_{-i}) \propto \frac{n_{w_i}^{(z_i)} + \beta}{n_{\bullet}^{(z_i)} + W\beta} \frac{n_j^{(d_i)} + \alpha}{n_{\bullet}^{(d_i)} + T\alpha}$$

- This is nicer than your average Gibbs sampler:
  - memory: counts can be cached in two sparse matrices
  - optimization: no special functions, simple arithmetic
  - the distributions on Φ and Θ are analytic given z and w, and can later be found for each sample

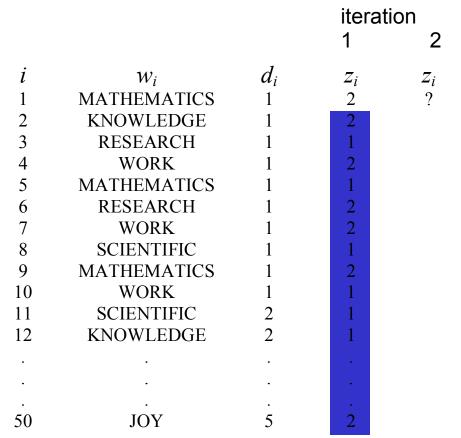
iteration

			1
i	$\mathcal{W}_{i}$	$d_i$	$Z_i$
1	MATHEMATICS	1	2
2	KNOWLEDGE	1	2
3	RESEARCH	1	1
4	WORK	1	2
5	MATHEMATICS	1	1
6	RESEARCH	1	2
7	WORK	1	2
8	SCIENTIFIC	1	1
9	MATHEMATICS	1	2
10	WORK	1	1
11	SCIENTIFIC	2	1
12	KNOWLEDGE	2	1
•			
•		•	•
50	IOY	5	2
20	<i>3</i> <b>(</b> 1	0	-

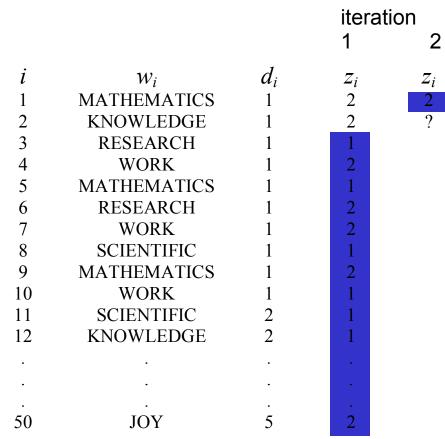
			iterat	ion
			1	2
i	${\mathcal W}_i$	$d_i$	$Z_i$	$Z_i$
1	MATHEMATICS	1	2	?
2	KNOWLEDGE	1	2	
3	RESEARCH	1	1	
4	WORK	1	2	
5	MATHEMATICS	1	1	
6	RESEARCH	1	2	
7	WORK	1	2	
8	SCIENTIFIC	1	1	
9	MATHEMATICS	1	2	
10	WORK	1	1	
11	SCIENTIFIC	2	1	
12	KNOWLEDGE	2	1	
•				
•		•	•	
•		•		
50	JOY	5	2	

			iterati	ion
			1	2
i	${\mathcal W}_i$	$d_i$	$Z_i$	$Z_i$
1	MATHEMATICS	1	2	?
2	KNOWLEDGE	1	2	
3	RESEARCH	1	1	
4	WORK	1	2	
5	MATHEMATICS	1	1	
6	RESEARCH	1	2	
7	WORK	1	2	
8	SCIENTIFIC	1	1	
9	MATHEMATICS	1	2	
10	WORK	1	1	
11	SCIENTIFIC	2	1	
12	KNOWLEDGE	2	1	
•				
•	•	•	•	
50	JOY	5	. 2	

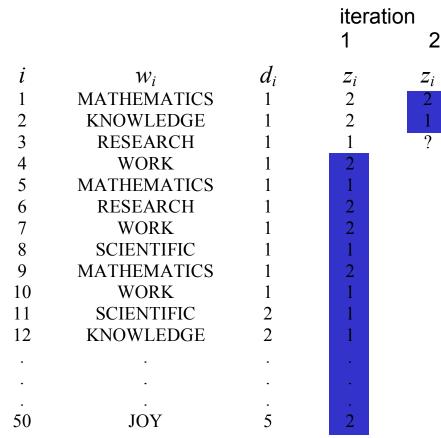
$$P(z_i=j|\mathbf{z}_{-i},\mathbf{w}) \propto rac{n_{-i,j}^{(w_i)}+eta}{n_{-i,j}^{(\cdot)}+Weta}rac{n_{-i,j}^{(d_i)}+lpha}{n_{-i,\cdot}^{(d_i)}+Tlpha}$$



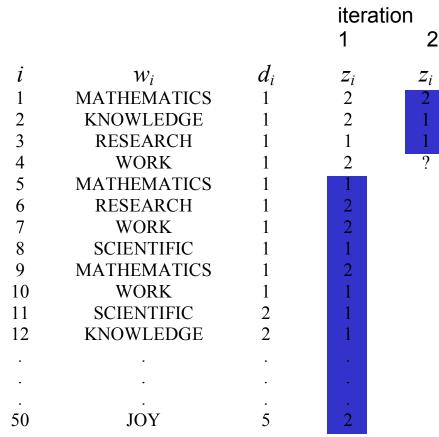
$$P(z_i=j|\mathbf{z}_{-i},\mathbf{w}) \propto rac{n_{-i,j}^{(w_i)}+eta}{n_{-i,j}^{(\cdot)}+Weta}rac{n_{-i,j}^{(d_i)}+lpha}{n_{-i,\cdot}^{(d_i)}+Tlpha}$$



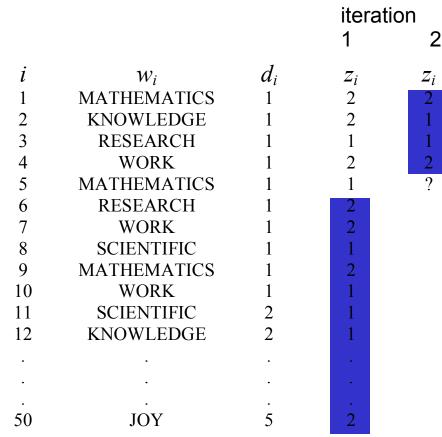
$$P(z_i=j|{f z}_{-i},{f w}) \propto rac{n_{-i,j}^{(w_i)}+eta}{n_{-i,j}^{(\cdot)}+Weta} rac{n_{-i,j}^{(d_i)}+lpha}{n_{-i,\cdot}^{(d_i)}+Tlpha}$$



$$P(z_i=j|{f z}_{-i},{f w}) \propto rac{n^{(w_i)}_{-i,j}+eta}{n^{(\cdot)}_{-i,j}+Weta} rac{n^{(d_i)}_{-i,j}+lpha}{n^{(d_i)}_{-i,\cdot}+Tlpha}$$



$$P(z_i=j|\mathbf{z}_{-i},\mathbf{w}) \propto rac{n_{-i,j}^{(w_i)}+eta}{n_{-i,j}^{(\cdot)}+Weta}rac{n_{-i,j}^{(d_i)}+lpha}{n_{-i,\cdot}^{(d_i)}+Tlpha}$$

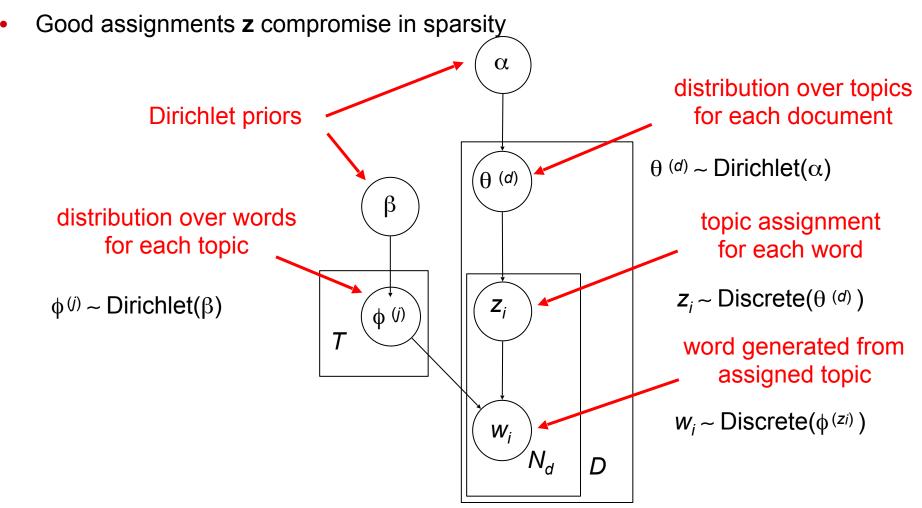


$$P(z_i=j|{f z}_{-i},{f w}) \propto rac{n^{(w_i)}_{-i,j}+eta}{n^{(\cdot)}_{-i,j}+Weta} rac{n^{(d_i)}_{-i,j}+lpha}{n^{(d_i)}_{-i,\cdot}+Tlpha}$$

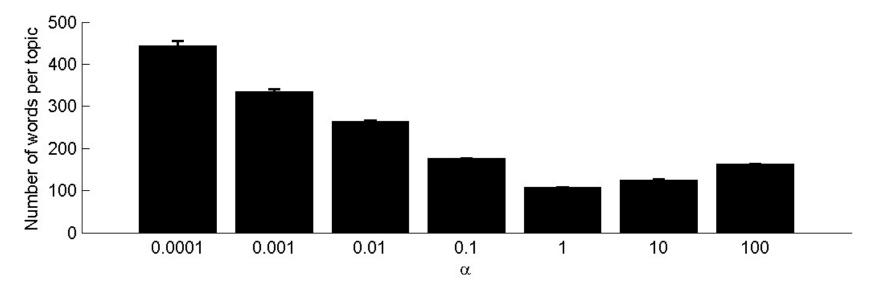
			itera	ation	
			1	2	1000
i	${\mathcal W}_i$	$d_i$	$Z_i$	$Z_i$	$Z_i$
1	MATHEMATICS	1	2	2	2
2	KNOWLEDGE	1	2	1	2
3	RESEARCH	1	1	1	2
4	WORK	1	2	2	1
5	MATHEMATICS	1	1	2	2
6	RESEARCH	1	2	2	2
7	WORK	1	2	2	2
8	SCIENTIFIC	1	1	1	1
9	MATHEMATICS	1	2	2	2
10	WORK	1	1	2	2
11	SCIENTIFIC	2	1	1	2
12	KNOWLEDGE	2	1	2	2
•		•	•		·
•	•	•	•	•	
50	JOY	5	2	1	1
				$P(z_i=j $	$({f z}_{-i},{f w}) \propto rac{n^{(w_i)}_{-i,j} + eta}{n^{(\cdot)}_{-i,j} + Weta} rac{n^{(d_i)}_{-i,j} + lpha}{n^{(d_i)}_{-i,\cdot} + Tlpha}  ,$

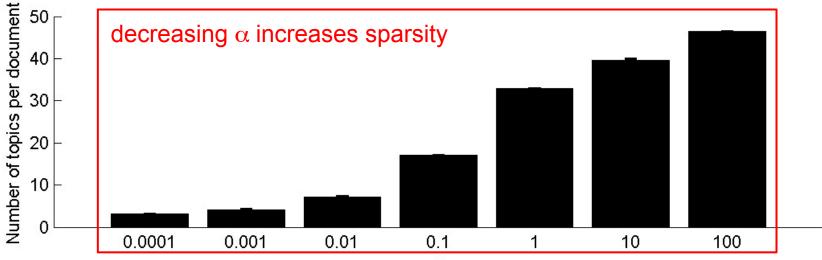
# Effects of hyperparameters

- $\alpha$  and  $\beta$  control the relative sparsity of  $\Phi$  and  $\Theta$ 
  - smaller  $\alpha$ , fewer topics per document
  - smaller  $\beta$ , fewer words per topic

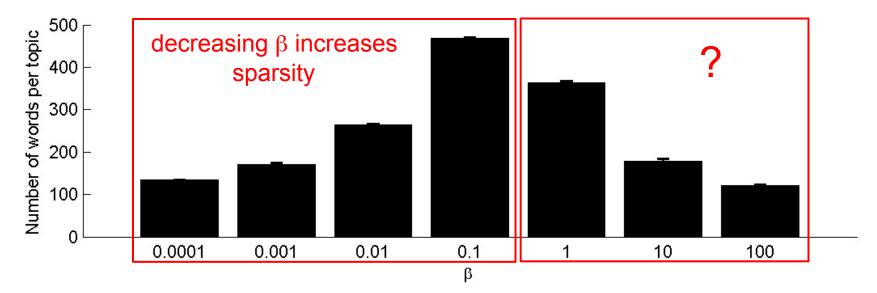


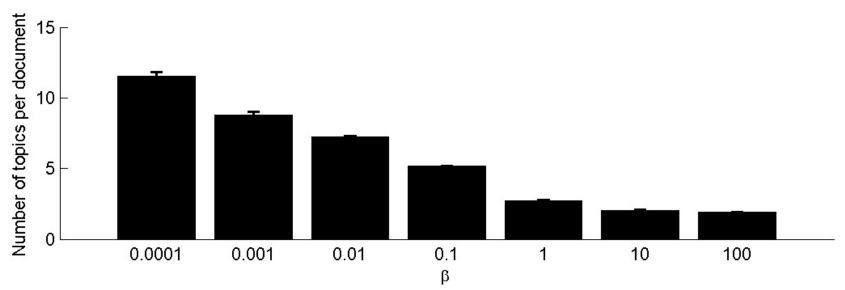
# Varying $\boldsymbol{\alpha}$

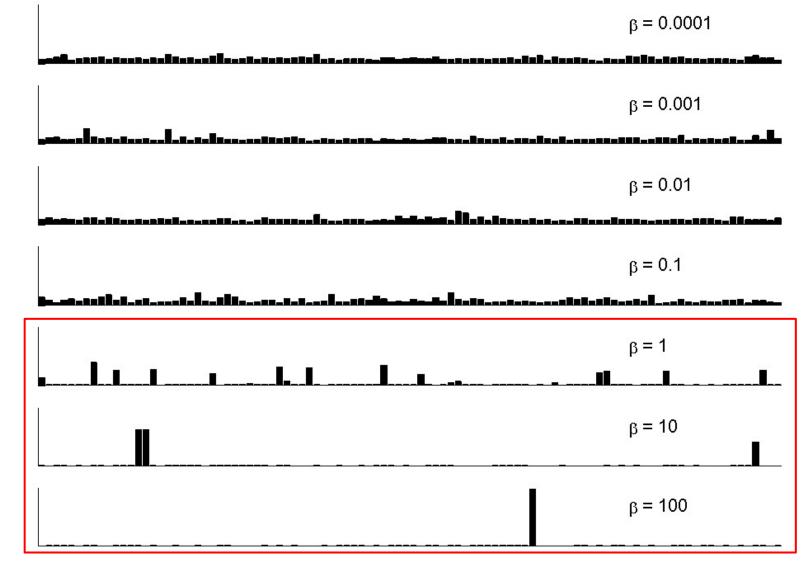




# Varying $\beta$







Topic

### Learning the number of topics

- Can use standard Bayes factor methods to evaluate models of different dimensionality
- Alternative: nonparametric Bayes
  - fixed number of topics per document, unbounded number of topics per corpus (Blei, Griffiths, Jordan, & Tenenbaum, 2004)
  - unbounded number of topics for both (the hierarchical Dirichlet process)

(Teh, Jordan, Beal, & Blei, 2004)

## Analysis of PNAS abstracts

- Test topic models with a real database of scientific papers from PNAS
- All 28,154 abstracts from 1991-2001
- All words occurring in at least five abstracts, not on "stop" list (20,551)
- Total of 3,026,970 tokens in corpus

(Griffiths & Steyvers, 2004)

### A selection of topics

FORCE SURFACE MOLECULES SOLUTION SURFACES MICROSCOPY WATER FORCES PARTICLES STRENGTH POLYMER IONIC ATOMIC AQUEOUS MOLECULAR PROPERTIES LIQUID SOLUTIONS BEADS **MECHANICAL** 

HIV VIRUS INFECTED **IMMUNODEFICIENCY** CD4 **INFECTION** HUMAN VIRAL TAT GP120 REPLICATION TYPE **ENVELOPE** AIDS REV BLOOD CCR5 **INDIVIDUALS** ENV PERIPHERAL

MUSCLE CARDIAC HEART SKELETAL **MYOCYTES** VENTRICULAR MUSCLES SMOOTH **HYPERTROPHY** DYSTROPHIN HEARTS CONTRACTION **FIBERS** FUNCTION TISSUE RAT **MYOCARDIAL ISOLATED** MYOD FAILURE

STRUCTURE ANGSTROM **CRYSTAL** RESIDUES STRUCTURES STRUCTURAL RESOLUTION HELIX THREE HELICES DETERMINED RAY CONFORMATION HELICAL **HYDROPHOBIC** SIDE DIMENSIONAL INTERACTIONS MOLECULE SURFACE

**NEURONS** BRAIN CORTEX CORTICAL **OLFACTORY** NUCLEUS **NEURONAL** I AYFR RAT NUCLEI CEREBELLUM CEREBELLAR LATERAL CEREBRAL LAYERS GRANULE LABELED **HIPPOCAMPUS** AREAS THALAMIC

TUMOR CANCER TUMORS HUMAN CELLS BREAST MELANOMA GROWTH CARCINOMA PROSTATE NORMAL CFLL METASTATIC MALIGNANT LUNG CANCERS MICE NUDE PRIMARY **OVARIAN** 

# Software

- MALLET (java)
- in R: topicmodels and Ida packages
- Ida (python)
- LDAvis (R)
- ... (lots more!)

### Web demo

http://cpsievert.github.io/LDAvis/reviews/reviews.html